Erratum in
Gaussian Process Optimization with Mutual Information

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Abstract
This paper describes a mistake in the article “Gaussian Process Optimization with Mutual Information”, Proceedings of the 31st International Conference on Machine Learning, JMLR, pp.253-261, 2014. It appears that the information given to the algorithm is not sufficient for the main theorem to hold true. The theoretical guarantees would remain valid in a setting where the algorithm observes the instantaneous regret instead of noisy samples of the unknown function.

Introduction. In our paper “Gaussian Process Optimization with Mutual Information” [1], we analyze an algorithm for sequential global optimization using Gaussian processes and we aim at proving upper bounds on the cumulative regret incurred by the algorithm. We found an error in the proof of Lemma 1, which invalidates the main theorem.

Notations. Let $f : \mathcal{X} \rightarrow \mathbb{R}$ be the unknown function to be optimized, which is a sample from a Gaussian process. Let’s fix $x^*, x_1, \ldots, x_T \in \mathcal{X}$ and the observations $y_i = f(x_i) + \epsilon_i$ where the noise variables $\epsilon_i$ are independent Gaussian noise $\mathcal{N}(0, \sigma^2)$. We define the instantaneous regret $r_t = f(x^*) - f(x_t)$ and,

$$M_T = \sum_{t=1}^{T} \left( r_t - \mathbb{E}[r_t \mid y_1, \ldots, y_{t-1}] \right).$$

Erratum. In Lemma 1, we claimed that $M_T$ is a Gaussian martingale with respect to $Y_T = y_1, \ldots, y_T$. Even if $M_t - M_{t-1}$ is a centered Gaussian conditioned on $Y_{T-1}$, it is wrong to say that $M_T$ is a martingale since it is not measurable with respect to $Y_T$. 

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Correction and consequences. In order to fix Lemma 1, it is possible to modify $M_T$ and use its natural filtration $\mathcal{F}_T = \{r_t\}_{t \leq T}$ instead of $Y_T$,

$$M_T = \sum_{t=1}^{T} \left( r_t - \mathbb{E}[r_t \mid \mathcal{F}_{t-1}] \right).$$

Then $M_T$ is a Gaussian martingale with respect to $\mathcal{F}_T$. Now to adapt the algorithm for this new quantity it needs to observe $r_t$ instead of $y_t$ to be able to compute both the posterior expectation and variance for all $x$ in $X$:

$$\mu_t(x) = \mathbb{E}[f(x) \mid \mathcal{F}_{t-1}] \text{ and } \sigma_t^2(x) = \text{Var}[f(x) \mid \mathcal{F}_{t-1}].$$

Comments on the experiments. We remark that the experiments performed in [1] are remarkably good in spite of Lemma 1 being unproved. After having discovered the mistake we were able to build scenarios were the GP-MI algorithm is overconfident and misses the optimum of $f$, and therefore incurs a linear cumulative regret.

References