Tree Automata and Applications: TD2
Tree homomorphisms

Emile Contal
http://econtal.perso.math.cnrs.fr/teaching

October 4, 2013

Exercise 1. Let $\mathcal{F} = \{f(,), g(), a\}$ and $\mathcal{F}' = \{f'(,), a\}$. Let us consider the tree homomorphism $h : T(\mathcal{F}) \to T(\mathcal{F}')$ determined by $h_\mathcal{F}$ defined by: $h_\mathcal{F}(f) = f'(x_1, x_2)$, $h_\mathcal{F}(g) = f'(x_1, x_1)$, and $h_\mathcal{F}(a) = a$. Is $h(T(\mathcal{F}))$ recognizable?

Let $L_1 = \{g^i(a) \mid i \geq 0\}$, then L_1 is a recognizable tree language, is $h(L_1)$ recognizable?

Let L_2 be the recognizable tree language defined by $L_2 = \mathcal{L}(\mathcal{A})$ where $\mathcal{A} = (\mathcal{Q}, \mathcal{F}, \mathcal{Q}_f, \Delta)$ is defined by: $\mathcal{Q} = \{q_a, q_g, q_f\}$, $\mathcal{Q}_f = \{q_f\}$, and Δ is the following set of transition rules:

$$\left\{ \begin{array}{cc} a \to q_a & g(q_a) \to q_g \\ f(q_a, q_a) \to q_f & f(q_g, q_g) \to q_f \\ f(q_a, q_g) \to q_f & f(q_g, q_a) \to q_f \\ f(q_a, q_f) \to q_f & f(q_f, q_a) \to q_f \\ f(q_g, q_f) \to q_f & f(q_f, q_g) \to q_f \\ f(q_f, q_f) \to q_f & \end{array} \right\} .$$

Is $h(L_2)$ recognizable?

Exercise 2. Let E be an non-empty subset of $T(\mathcal{F}_1)$ and $\mathcal{F}_2$ be any finite alphabet. Prove that it exists a tree homomorphism $\pi : T(\mathcal{F}_2) \to T(\mathcal{F}_1)$ such that $\pi^{-1}(E) = T(\mathcal{F}_2)$.

Let $\mathcal{F}_n$ be an alphabet with symbol of arity at most n, and L be a recognizable tree language on $\mathcal{F}_n$ and $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = L$. Prove that it exists two linear homomorphisms $h_\mathcal{A}$ and d such that:

$$\begin{aligned} h_\mathcal{A} &: T(\mathcal{F}_n \times \mathcal{Q}^{n+1} \cup \mathcal{Q}) \to T(\mathcal{F}_n) \\ d &: T(\mathcal{F}_n \times \mathcal{Q}^{n+1} \cup \mathcal{Q}) \to T(\mathcal{F}_n) \ , \end{aligned}$$

and $d\big(h_\mathcal{A}^{-1}(T(\mathcal{F}_n))\big)$ is the set of accepting computations of $\mathcal{A}$.

Prove that the class of recognizable tree languages is the smallest non trivial class of tree languages closed by linear tree homomorphisms and inverse tree homomorphisms.