Can Small Islands Protect Nearby Coasts from Tsunamis? An active experimental design approach

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Run-up Amplification: Stefanakis et al. (2011)

Local Run-Up Amplification by Resonant Wave Interactions

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more light on the resonant mechanism ($\omega = 0.4$ s$^{-1}$, $\tan \theta = 0.13$). The first snapshot is taken at the instant
Tsunamis Amplification Phenomenon

2010 Sumatra tsunami and the Mentawai Islands (Hill et al., 2012)
### Numerical Simulations

Adaptive mesh grid of the VOLNA solver
Tsunamis Amplification Phenomena

Numerical simulations of a tsunami amplification generated by a conical island

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Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan \theta_i$</td>
<td>$0.05 - 0.2$</td>
</tr>
<tr>
<td>$\tan \theta_b$</td>
<td>$0.05 - 0.2$</td>
</tr>
<tr>
<td>$d$</td>
<td>$0 - 5000m$</td>
</tr>
<tr>
<td>$h$</td>
<td>$100 - 1000m$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$0.01 - 0.1 \text{rad/s}$</td>
</tr>
</tbody>
</table>

Five parameters modelling the geometry
Problem Statement

Setup

- $d$ real parameters denoted by $d$-dimensional vectors $x \in \mathcal{X}$
- $\mathcal{X} \subseteq \mathbb{R}^d$ compact and convex
- Unknown objective function $f(x) \in \mathbb{R}$ for all $x \in \mathcal{X}$
- Noisy measurement $y = f(x) + \epsilon$, where $\epsilon \overset{iid}{\sim} \mathcal{N}(0, \eta^2)$

Goal
Find the parameters $x$ maximizing $f(x)$
Constraints

Challenges

- Expensive evaluations
- Joint optimization of several parameters

Case: run-up

- 5 parameters
- Each simulation takes 2 hours of computation
- A regular grid with 10 values per parameters needs $10^5$ points
- The standard approach would take 23 years of computation
Motivating Example: Sequential Optimization

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Motivating Example: Sequential Optimization

Parameter vs Objective

(x_1, y_1)
(x_2, y_2)
(x_3, y_3)
(x_4, y_4)
(x_5?)

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Motivating Example: Batch Optimization

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### Objective

**Setting**

After \( t \) iterations of the sequential query procedure,

- Query the batch \( x_{t+1}^1, \ldots, x_{t+1}^K \) using the information acquired during the previous iterations
- Observe the respective noisy evaluations \( y_{t+1}^1, \ldots, y_{t+1}^K \)

**Target**

- We want \( \max_{1 \leq t \leq T} \max_{1 \leq k \leq K} f(x_t^k) \xrightarrow{T \to \infty} \max_{x \in X} f(x) \) as fast as possible.
- Exploration vs Exploitation tradeoff

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Gaussian Processes: Framework

Definition

\( f \sim \mathcal{GP}(m, k) \), with mean function \( m : \mathcal{X} \rightarrow \mathbb{R} \) and covariance function \( k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}^+ \), when for all \( x_1, \ldots, x_n \),

\[
(f(x_1), \ldots, f(x_n)) \sim \mathcal{N}(\mu, K),
\]

with \( \mu[x_i] = m(x_i) \) and \( K[x_i, x_j] = k(x_i, x_j) \).

Probabilistic smoothness assumption

- Nearby locations are highly correlated
- Large local variations have low probability

Example of covariance function

- Squared Exponential RBF: \( k(x, y) = \exp(-\frac{\|x-y\|^2}{2\ell^2}) \)
- Rational Quadratic: \( k(x, y) = (1 + \frac{\|x-y\|^2}{2\alpha\ell^2})^{-\alpha} \)
Gaussian Processes: Examples

1D Gaussian Processes with different covariance functions

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Gaussian Process Interpolation

Bayesian Inference (Rasmussen and Williams, 2006)

At iteration $t$, with observations $Y_t$ for the query points $X_t$, the posterior mean and variances are given at all point $x$ in the search space by:

$$
\mu_t(x) := \mathbb{E}[f(x) | X_t, Y_t] = k_t(x) \mathbf{C}_t^{-1} Y_t 
$$

(1)

$$
\sigma^2_t(x) := \nabla[f(x) | X_t, Y_t] = k(x, x) - k_t(x) \mathbf{C}_t^{-1} k_t(x),
$$

(2)

where $\mathbf{C}_t = \mathbf{K}_t + \eta^2 \mathbf{I}$ and $\mathbf{K}_t = [k(x_t, x_{t'})]_{x_t, x_{t'} \in X_t}$.

Interpretation

- posterior mean $\mu_t$: prediction
- posterior variance $\sigma^2_t$: uncertainty
Gaussian Process Interpolation: Example

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Upper and Lower Confidence Bounds

Definition
Fix $0 < \delta < 1$,

\[ f_t^+(x) = \mu_t(x) + \sqrt{\beta_t \sigma_t^2(x)} \]
\[ f_t^-(x) = \mu_t(x) - \sqrt{\beta_t \sigma_t^2(x)} \]

with $\beta_t = \mathcal{O}(\log \frac{t}{\delta})$ defined in Srinivas et al. (2012)

Property

\[ \forall x \in \mathcal{X}, \forall t \geq 1, \]
\[ f(x) \in [f_t^-(x), f_t^+(x)] \text{ holds with probability at least } 1 - \delta \]
Relevant Region $\mathcal{R}_t$
The GP-UCB-PE algorithm (Contal et al., 2013)

\[ x_t^1 = ? \]
\[ x_t^2 = ? \]
\[ x_t^3 = ? \]
The GP-UCB-PE algorithm (Contal et al., 2013)

\[ x_t^1 = \arg\max_{x \in \mathcal{R}_t} f_t^+(x) \]

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Theoretical Analysis

**Theorem (Contal et al. (2013))**

With $f \sim \mathcal{GP}(0, k)$, with probability at least $1 - \delta$:

$$\max_{x \in X} f(x) - \max_{1 \leq t \leq T, 1 \leq k \leq K} f(x_t^k) = O\left(\sqrt{\frac{\gamma_{TK}}{TK}}\right)$$

**Information theory**

$\gamma_{TK}$ is the maximum information gain about $f$ obtainable by a sequence of $TK$ queries.

- For linear covariance, $\gamma_{TK} = O(d \log TK)$
- For Squared Exponential covariance, $\gamma_{TK} = O\left((\log TK)^{d+1}\right)$
Batch vs Sequential

**Complexity**

If $n$ is the number of training points,

Sequential: $n \text{Cost}(f) + n \text{Cost}(\text{GP})$

Batch: $\frac{n}{K} \text{Cost}(f) + n \text{Cost}(\text{GP})$

With exact inference, $\text{Cost}(\text{GP}) = \mathcal{O}(n^2)$.

**Impact on the convergence speed**

Take $K \ll T$, then for equivalent $\text{Cost}(f)$ the improvement of the parallel strategy over the sequential one is $\sqrt{K}$ with respect to the convergence speed.
Histogram of the run-up amplification

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Local sensitivity of the maximum run-up amplification

(a) $RA$ vs. $\tan \theta_i$ and $\tan \theta_b$

(b) $h$ vs. time

(c) $RA$ vs. $d$

(d) $\omega$ vs. time

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